arXiv:hep-th/9410044v1 7 Oct 1994

FREE FIELDS

FOR ANY SPIN IN RELATIVISTIC

QUANTUM MECHANICS

D. R. Grigore[1]

Dept. Theor. Phys., Inst. Atomic Phys.,
Bucharest-Măgurele, P. O. Box MG 6, ROMÂNIA

Abstract

A general discussion of the construction of free fields based on Weinberg anszatz is provided and various applications appearing in the litterature are considered.

---

[1]e-mail: grigore@roifa.bitnet, grigore@ifa.ro

1   Introduction

In this letter we provide a general discussion of the procedure of constructing a relativistic free quantum field when it is required that the one-particle Hilbert space carries a certain projective unitary representation of some invariance group. The usual case is when the invariance group is the Poincaré group and the representation is irreducible, corresponding to an elementary system.

The discussion was initiated a long time ago by Weinberg [1]-[4] who provided a rather exhaustive analysis of the usual case when the fields are living in the four-dimensional Minkowski space. Recently the analysis was extended to the three-dimensional case with the purpose of obtaining free fields for any spin [5].

We will show that a general discussion of this problem is possible. One recovers extremely easy the results alluded above and, moreover one can treat other types of fields which describe excitations which are not point-like. We have in mind, for instance, fields localized on cones which appear in three-dimensional quantum fields theory (see for instance [6].)

The general theory will be developped in Section 2 and the applications in Section 3.

2   Relativistic Quantum Field Theory

2.1 We begin with a general discussion of the notion of relativistic quantum field theory. As it is well known [7], [8], [9] a quantum field theory is an ansamble $(\mathcal{H}, \Phi_0, D, \phi_\alpha(x))$ where $\mathcal{H}$ is a complex Hilbert space, Φ_0 is the vacuum state, $D \in \mathcal{H}$ is a dense domain containing Φ_0 and $\phi_\alpha(x)$ are some distribution-valued operators such that $\phi_\alpha(x)$ and $\phi_\alpha(x)^*$ are defined on D and are leaving it invariant. Here x takes values in some configuration manifold X, which is usually the Minkowski space. For a relativistic quantum field one supposes moreover that an invariance group G_{inv} is given; G_{inv} acts on the manifold X and according to the basic principles of quantum theory [10] we also have an projective unitary representation of G_{inv} in $\mathcal{H}$. We will consider the usual case when this representation follows from a true representation $G \ni g \mapsto \mathcal{W}_g \in \mathcal{U}(\mathcal{H})$ of the universal covering group G of G_{inv}; here $\mathcal{U}(\mathcal{H})$ is the set of the unitary operators in $\mathcal{H}$. The general case can be also treated on similar lines. The interpretation of these objects being obvious, one can argue that the fields $\phi_\alpha(x)$ should behave as follows:

$$\mathcal{W}_g \phi_\alpha(x) \mathcal{W}_g^{-1} = c_{\alpha\beta}(g^{-1}, g \cdot x)\phi_\beta(g \cdot x) \tag{1}$$

We use the summation convention over the dummy indices. Here $c_{\alpha\beta}(g, x)$ is a matrix-valued function defined on $G \times X$ with complex entries. The indices α, β take values $1, 2, ..., M$ where M is integer or even infinite.

From the previous equation one easily obtains a consistency condition:

$$c(g_1, g_2 \cdot x)c(g_2, x) = c(g_1 g_2, x) \tag{2}$$

i.e in the cohomology terminology [10], c is a $(G, X, \mathcal{K})$-cocycle. Here $\mathcal{K} = GL(M, \mathbf{C})$.

Moreover, if we have two cohomologous cocycles, $c^{(1)}$ and $c^{(2)}$ i.e there exists a map $b : X \mapsto \mathcal{K}$ such that

$$c^{(2)}(g, x) = b(g \cdot x)c^{(1)}(g, x)b(x)^{-1} \tag{3}$$

and the field $\phi^{(1)}$ verifies (1) with $c = c^{(1)}$, let us define

$$\phi_\alpha^{(2)}(x) = b_{\alpha\beta}\phi_\beta^{(1)}(x). \tag{4}$$

Then $\phi^{(2)}$ verifies (1) with $c = c^{(2)}$.

This argument shows that in order to classify all possible relativistic laws of transformation of the type (1) it is sufficient to take a representative cocycle c from every cohomology class. According to the general theory from [10], this analysis is possible if G acts transitively on X. Then the cohomology classes are in one-one correspondence with the equivalence classes of representation of the stability group of some point $x_0 \in X$.

The case when G admits non-trivial multiplicators can be analysed on similar lines. The relation (2) will be true up to some phase factor depending on g_1 and g_2.

2.2 A particular important case is when $X = M_N$ (i.e. the N-dimensional Minkowski space) and $G = \widetilde{\mathcal{P}_+^\uparrow} = H \times_t A$ with A the translation group of M_N, and $H = \widetilde{\mathcal{L}_+^\uparrow}$. The action of G on X is

$$(h, a) \cdot x = h \cdot x + a. \tag{5}$$

This action is transitive and taking $x_0 = 0$ we have $G_0 = \{(h, 0)|h \in H\} \simeq H$. So, the the cohomology classes of cocycles c are indexed by equivalence classes of representations of H. Let $h \mapsto D(h)$ be such a representation of H in $\mathbf{C}^M$; D is not necessarely unitary. Then the cocycle induced by D is simply

$$c^D((h, a), x) = D(h). \tag{6}$$

So, (1) takes the usual form (in matrix notations):

$$\mathcal{W}_{h,a}\phi(x)\mathcal{W}_{h,a}^{-1} = D(h^{-1})\phi(h \cdot x + a). \tag{7}$$

We note in closing this subsection that one usually supposes that on X there exists a supplementary structure of causality which is G-invariant and then one requires that $\phi_\alpha(x)$ commute or anticommute with $\phi_\beta(y)$ and $\phi_\beta(y)^*$ for x and y causally separated.

2.3 Let us suppose now that, in the framework of the preceeding subsection, $\phi_\alpha(x)$ is a free field. Then $\mathcal{H}$ is a Fock space $\mathcal{H} = \oplus_{n=0}^\infty \mathcal{H}^{(n)}$ with annihilation and creation operators $a(f)$, $a^*(f)$, $(\forall f \in \mathcal{H}^{(1)})$ [11].

Then (7) is true for positive and negative frequency parts $\phi_\alpha^{(\pm)}(x)$ i.e. we have:

$$\mathcal{W}_{h,a}\phi^{(\pm)}(x)\mathcal{W}_{h,a}^{-1} = D(h^{-1})\phi^{(\pm)}(h \cdot x + a). \tag{8}$$

2

We make now the *Weinberg anszatz*, namely:

$$\phi^{(+)}(x) = a(f_{\alpha,x}) \tag{9}$$

where $f_{\alpha,x}$ are some generalized function from the one-particle Hilbert space $\mathcal{H}^{(1)}$.

It is well known [10] that $\mathcal{H}^{(1)}$ can be exhibited in the form

$$\mathcal{H}^{(1)} = \{\Phi : X_m^+ \mapsto \mathbf{C}^M | \int_{X_m^+} d\alpha_m^+ |\Phi(p)|^2 < \infty\} \tag{10}$$

and the representation of G in $\mathcal{H}^{(1)}$ of the form:

$$(W_{h,a}\Phi)_i(p) = e^{ip \cdot x} C(h, h^{-1} \cdot p)_{ij} \Phi_j(h^{-1} \cdot p). \tag{11}$$

Here C is a $(H, X_m^+, \mathbf{C}^M)$-cocycle; the formulas above are known under the name of Wigner formalism. The representation $\mathcal{W}$ acting in $\mathcal{H}$ is given by:

$$\mathcal{W}_{h,a} \equiv \Gamma(W_{h,a}). \tag{12}$$

Then one knows that:

$$\mathcal{W}_{h,a} a(f) \mathcal{W}_{h,a}^{-1} = a(W_{h,a} f). \tag{13}$$

Also one has:

$$(a(f)\Phi)_{i_1,\dots,i_n}^{(n)}(p_1, \dots, p_n) = \sqrt{n+1} \int_{X_m^+} d\alpha_m^+(p_0) \overline{f_{i_0}(p_0)} \Phi_{i_0,\dots,i_n}(p_0, \dots, p_n). \tag{14}$$

So, one can insert everything in (8) and obtain immediately:

$$W_{h,a} f_{\alpha,x} = \overline{D_{\alpha\beta}(h^{-1})} f_{\beta, h \cdot x + a}. \tag{15}$$

From this relation it easily follows that $f_{\alpha,x}$ is of the form:

$$f_{\alpha,x}(p) = e^{ip \cdot x} f_\alpha(p) \tag{16}$$

where f_α verifies the relation:

$$f_\alpha(h \cdot p) = \overline{D_{\alpha\beta}(h)} C(h, p) f_\beta(p). \tag{17}$$

It is known [10] that the cocycle C is induced, in the sense of Mackey, by some unitary representation π of the "little group" H_{p_0}, $(p_0 \in X_m^+)$ given by:

$$\pi(h_0) \equiv C(h_0, p_0), \quad (\forall h_0 \in H_{p_0}). \tag{18}$$

Then (17) implies that

$$f_\alpha(p_0) = \overline{D_{\alpha\beta}(h)} \pi(h_0) f_\beta(p_0), \quad (\forall h_0 \in H_{p_0}). \tag{19}$$

3

If $f_\alpha(p_0)$ is a solution of this equation, then we have from (17) that:

$$f_\alpha(p) = \overline{D_{\alpha\beta}}(\sigma(p))C(\sigma(p), p_0)f_\beta(p_0) \tag{20}$$

where $\sigma : X_m^+ \mapsto H$ is a Borel cross section, i.e. we have $\sigma(p_0) = e_H$ and $\sigma(p)\cdot p_0 = p$.

It follows that the key relation is (19) which says that $f_{\alpha,i}(p_0)$ is an invariant vector for the representation $\overline{D} \otimes \pi$ of H_{p_0}. This is possible *iff* the Clebsh-Gordan series of $\overline{D} \otimes \pi$ contains the trivial representation.

The same argument goes for

$$\phi_\alpha^{(-)}(x) = a^*(g_{\alpha,x}) \tag{21}$$

with the result that $\overline{D} \otimes \overline{\pi}$ should also contain the trivial representation. So we have:

Theorem 1: *A necessary and sufficient condition for the existence of the free field verifying (7) where W is given by (12), is that the Clebsh-Gordan series of $D \otimes \pi$ and $D \otimes \overline{\pi}$ should contain the trivial representation. In this case the free field is given by the formulas (9)+(21) where $f_{\alpha,x}$ and $g_{\alpha,x}$ are given by formulas of the type (20).*

Proof: One has to show that the analysis above does not depend on the explicit realization of $\mathcal{H}^{(1)}$ and W. (One could exhibit for instance W in the Hilbert space fiber bundle formalism [10].) But one knows that if $\mathcal{H}'^{(1)}$ and W' is another representation of G induced also by π, then there exists an unitary operator $V : \mathcal{H}'^{(1)} \mapsto \mathcal{H}^{(1)}$ intertwining W and W'. One extends V to the whole Hilbert space in a natural way, obtains a relation of the type (15) with $f_{\alpha,x} \mapsto V f_{\alpha,x}$ and the previous analysis remains unchanged. $\square$

Remarks

1) Let us suppose now that $D^{(1)}$ and $D^{(2)}$ are two representations of H_{p_0} admitting the direct sum decomposition:

$$D^{(i)} = \oplus_{\alpha_i \in I_i} \pi^{(\alpha_i)}, \quad (i = 1, 2) \tag{22}$$

for some index sets I_i. Then:

$$D^{(1)} \otimes D^{(2)} = \sum_{\alpha_1 \in I_1, \alpha_2 \in I_2} \pi^{(\alpha_1)} \otimes \pi^{(\alpha_2)} \tag{23}$$

so $D^{(1)} \otimes D^{(2)}$ contains the identity representation *iff* there exists $\alpha_1 \in I_1, \alpha_2 \in I_2$ such that $\pi^{(\alpha_1)} \otimes \pi^{(\alpha_2)}$ contains the trivial representation. Applying Schur lemma it follows that $(\pi^{(\alpha_1)})^t \simeq \pi^{(\alpha_2)}$ where $(\pi^{(\alpha)})_{ij}^t(h) \equiv \pi_{ji}^{(\alpha)}(h^{-1})$.

In our case $D^{(1)} \mapsto D|_{H_{p_0}}$ and $D^{(2)} \mapsto \pi, \overline{\pi}$. For an elementary particle, π is irreducible, so the criterion above sums up to the requirement that $D|_{H_{p_0}}$ should contain π^t and $\overline{\pi}^t$ as subrepresentations.

If we want that the equations of the type (19) have an unique solution, then these subrepresentations should appear only once.

2) If $N = M$, the preceeding remark implies that $D|_{H_{p_0}} \simeq \pi^t$ and $D|_{H_{p_0}} \simeq \overline{\pi}^t$ so π should be self-contragradient.

4

3 Applications

3.1 First, consider the case $N = 4$ i.e. the four-dimensional Minkowski space to the purpose of obtaining the results of Weinberg. One knows [10] that it is possible to take $H = SL(2, \mathbf{C})$. For particles of non-zero mass $m \in \mathbb{R}^*$, and spin $s \in \mathbf{Z}/2$ we have $H_{p_0} \simeq SU(2)$ and $\pi \simeq D^{(s)}$ (i.e. the irreducible representation of weight s of $SU(2)$.) As $D^{(s)}$ is self-contragradient, we are looking for a representation D of $SL(2, \mathbf{C})$ such that $D|_{H_{p_0}}$ contains $D^{(s)}$. If this subrepresentation is contained only once, then the free field is essentially unique. Such a representation is, in standard notations, $D^{(s,0)}$. The construction of the corresponding free field according to the scheme from subsection 2.3 gives us the results from [7], ch. 3-2. This is the result from [1].

For particles of zero mass $m = 0$ and helicity $n \in \mathbf{Z}/2$, one knows that $H_{p_0} \simeq SE(2)$ and $\pi \simeq \pi_n$ (see [10] ch. IX-3.) So, we are looking for a representation D of $SL(2, \mathbf{C})$ such that $\overline{D}|_{H_{p_0}}$ contains only once π_n and $\overline{\pi_n} \simeq \pi_{-n}$. Such a representation exists and it is $D^{(s_1, s_2)}$ with $s_1 - s_2 = n$. The same assertion is true if one takes $\pi = \pi_n \oplus \pi_{-n}$ as it is the case, for instance, for the photon. We have recovered the results of [2].

3.2 We consider now the case $N = 3$ i.e X is the three-dimensional Minkowski space. An explicit description of H and G can be found in [12]. For particles of non-zero mass $m \in \mathbb{R}^*$ and any spin $s \in \mathbb{R}$ we have $H_{p_0} \simeq \mathbb{R}$ and $\pi \simeq \pi^{(s)}$ (in the notations of [12].) Then one tries to find a representation D of H such that $\overline{D}|_{H_{p_0}}$ contais $\pi^{(s)}$ and $\overline{\pi^{(s)}}$. Such a representation have been exhibited in [5], [12] and is $D = T^{(s/2,+)} \oplus T^{(-s/2,+)}$ where $T^{(s,\pm)}$ are representations from the discrete series. For particles of zero-mass $m = 0$ we have $H_{p_0} \simeq \mathbf{Z} \times \mathbb{R}$ and $\pi \simeq \pi^{(s,t)}$ (see [12].) If $t = 0$ then one can take D as before. We have recovered the results of [5].

3.3 For completeness we also analyse the case $N = 1$ i.e. the two-dimensional Minkowski space (se also [13].) Explicitely, $G = \mathbb{R} \times \mathbb{R}^2$ with the composition law

$$(\chi_1, a_1) \cdot (\chi_2, a_2) = (\chi_1 + \chi_2, a_1 + \Lambda(\chi_1)a_1) \tag{24}$$

where

$$\Lambda(\chi) = \begin{pmatrix} ch(\chi) & sh(\chi) \\ sh(\chi) & ch(\chi) \end{pmatrix}.$$

So in this case, $H = \{(\chi, 0) | \chi \in \mathbb{R}\} \simeq \mathbb{R}$. The new technical element in analysing this case is that G although simply connected, admits non-trivial multiplicators. So we will have genuine projective representations. Using the usual canonical identification $H^2(G, \mathbb{R}) \simeq \mathrm{H}^2(\mathrm{Lie}(G), \mathbb{R})$ valid for G connected and simply connected [10] one discovers that every multiplier of G is cohomologous to the (non-trivial) multiplier m_k $(k \in \mathbb{R})$ given by:

$$m_k((\chi_1, a_1), (\chi_2, a_2)) = exp\left\{ i\frac{k}{2} < a_1, \Lambda(\chi_1)a_2 > \right\}, \tag{25}$$

where for any $a, b \in \mathbb{R}$, $< \cdot, \cdot >$ is the Lorentz invariant sesquilinear form:

$$< a, b > \equiv a_0 b_1 - a_1 b_0. \tag{26}$$

Then we have

Theorem 2: *Every m_k-projective unitary irreducible representation of G is unitary equivalent to one of the following form:*

(i) for $k \in \mathbb{R}^*, \mathrm{W}^{k,c}$ ($c \in \mathbb{R}$) acting in $L^2(\mathbb{R}, \mathrm{d}\lambda)$ according to:

$$\left(W_{0,a}^{k,c} f \right)(\lambda) = exp \left\{ i a_0 \left(\lambda + \frac{k}{2} a_1 \right) \right\} f(\lambda + k a_1) \tag{27}$$

$$W_{\chi,0}^{k,c} = e^{ic\chi} e^{i\chi \overline{B_0}} \tag{28}$$

where $B_0 \equiv -\frac{1}{2k} \left(k^2 \frac{\partial^2}{\partial \lambda^2} + \lambda^2 \right)$ is defined on $\mathcal{C}^\infty(\mathbb{R})$, and $\overline{B_0}$ is the (unique) self-adjoint extension of B_0 to $L^2(\mathbb{R}, \mathrm{d}\lambda)$.

(ii) for $k = 0$, we have:

(a) $W^{m,\eta}$ ($m \in \mathbb{R}_+ \cup \{0\}, \eta = \pm$) acting in $L^2(X_m^+, \mathrm{d}\alpha_m^+)$ acording to:

$$\left(W_{\chi,a}^{m,\eta} f \right)(p) = e^{ia \cdot p} \ f(\Lambda(\chi)^{-1} \cdot p). \tag{29}$$

(b) W^{im} ($m \in \mathbb{R}^*$) acting in $L^2(Y_m, \beta_m)$ acording to:

$$\left(W_{\chi,a}^{im} f \right)(p) = e^{ia \cdot p} \ f(\Lambda(\chi)^{-1} \cdot p). \tag{30}$$

(c) W^c ($c \in \mathbb{R}$) acting in $\mathbf{C}$ according to

$$W_{\chi,a}^c = e^{ic\chi} \tag{31}$$

Proof: The case $k = 0$ is standard and it is analysed as in [10]. We concentrate on the case $k \in \mathbb{R}^*$.

Let $W_{\chi,a}$ be a m_k-projective unitary irreducible representation of G. We define

$$U_a = W_{0,a}, \quad V_\chi = W_{\chi,0} \tag{32}$$

and we have:

$$V_\chi V_{\chi'} = V_{\chi+\chi'} \tag{33}$$

$$U_{a+a'} = exp \left\{ i \frac{k}{2} < a, a' > \right\} U_a U_{a'} \tag{34}$$

$$V_\chi U_a V_\chi^{-1} = U_{\Lambda(\chi)a}. \tag{35}$$

Conversely, if V_χ and U_a are (unitary) operators verifying (33)-(35) then:

$$W_{\chi,a} \equiv U_a V_\chi \tag{36}$$

6

is a m_k-projective unitary irreducible representation of G. We first analyse (34). It is easy to connect U_a to a representation of the Weyl canonical commutation relations (in general a reducible one.) Using the standard representation theorem of Stone-von-Neumann [10] we find out easily that one can take $\mathcal{H} = L^2(\mathbb{R}, d\lambda, \mathbf{C}^M)$ (where for the moment $M = 1, 2..., \infty$ is undetermined) and

$$(U_a f)(\lambda) = exp\left\{ia_0\left(\lambda + \frac{k}{2}a_1\right)\right\}f(\lambda + ka_1). \tag{37}$$

Next, one analyses the degree of arbitrariness of V_χ. If V_χ and V'_χ both verify (33) and (35) then $\mathcal{V}_\chi \equiv V_\chi^{-1}V'_\chi$ commutes with U_a, so, using lemma 6.4 from [10], one easily descovers that it has the form:

$$(\mathcal{V}_\chi f)(\lambda) = g_\chi f(\lambda) \tag{38}$$

with g_χ a $M \times M$ complex valued matrix.

From (33) it follows that in fact $g_\chi = e^{iC\chi}$ with C a real $M \times M$ self-adjoint matrix. So: $V'_\chi = e^{iC\chi}V_\chi$.

According to Stone theorem, $V_\chi = e^{i\chi B}$ with B an self-adjoint operator in $L^2(\mathbb{R}, d\lambda, \mathbf{C}^M)$. Using infinitesimal arguments, one discovers that a possible solution of (33)+(35) is $\overline{B_0}$ where $B_0 \equiv -\frac{1}{2k}\left(k^2\frac{\partial^2}{\partial\lambda^2} + \lambda^2\right).$

According to the argument above it follows that the most general B is $B = \overline{B_0}+C$ with C described previously.

Finally, one notices that, by a unitary transformation in $\mathcal{H}$, one can take C to be diagonal. If $M > 1$ then W given by (36) is reducible. So we must have $M = 1$.□

We can analyse now the existence of free fields. For the case $k = 0$ the analysis is elementary. In the case $k \neq 0$ we proceed as follows. First, one shows that although we have a genuine projective representation, relation (2) remains unchanged. So, again we obtain as a possible transformation law for the field a relation of the type (7). We need in the following only the behviour with respect to translation i.e. the case $h = e_H$:

$$\mathcal{W}_{e_H,a}\phi_\alpha(x)\mathcal{W}_{e_H,a}^{-1} = \phi_\alpha(x + a) \tag{39}$$

As before, we take $\mathcal{H}^{(1)} = L^2(\mathbb{R}, d\lambda)$ with the representation W given by (27)+(28), $\mathcal{H} = \mathcal{F}(\mathcal{H}^{(1)})$ with $\mathcal{W} = \Gamma(W)$ and $\phi_\alpha^{(\pm)}$ given by Weinberg anszatz (9)+(21). Then one easily finds out that $f_{\alpha,x}$ verifies:

$$f_{\alpha,x+a}(\lambda) = exp\left\{ia_0\left(\lambda + \frac{k}{2}a_1\right)\right\}f_{\alpha,x}(\lambda) \tag{40}$$

which has only the trivial solution $f_{\alpha,x} = 0$. So in this case we cannot construct free fields.

3.4 We will exhibit the flexibility of the formalism developped above by treating the case of a configuration space X which is not the Minkowski space. We consider

$$X \equiv \{(x,\nu) \in M_N \times M_N | \nu^2 = -1\} \tag{41}$$

where $\nu^2 = \nu_0^2 - \boldsymbol{\nu^2}$ is the Minkowski norm of ν.

This configuration space can be physically interpreted as describing cones of fixed angle [6]. Then x is the position of the apex and ν is giving the direction of the symmetry axis. (In the "rest frame" where $\nu = (0, \nu), \nu$ is precisely the direction of the symmetry axis.)

The action of the Poincaré group (and its universal covering group) on X is natural:

$$(h, a) \cdot (x, \nu) = (h \cdot x + a, h \cdot \nu). \tag{42}$$

So, now the fields are operators of the type $\phi_\alpha(x, \nu)$ and, instead of (7), we will have:

$$\mathcal{W}_{h,a}\phi(x, \nu)\mathcal{W}_{h,a}^{-1} = D(h^{-1})\phi(h \cdot x + a, h \cdot \nu) \tag{43}$$

It is natural to assume, instead of (9) something like:

$$\phi_\alpha^+(x, \nu) = a(f_{\alpha,x,\nu}). \tag{44}$$

Then the analysis of the subsection 2.3 can be followed practically unchanged. One gets (15) with obvious modifications. So we keep a relation of the type (16), but instead of (17) we have:

$$\phi_{\alpha,x,\nu}(h \cdot p) = \overline{D_{\alpha\beta}(h)}C(h, p)\phi_{\beta,x,\nu}(p). \tag{45}$$

We take here $p = p_0$, $\nu = \nu_\rho \equiv (\rho, 0, ..., 0, \sqrt{\rho^2 + 1})$, (where e.g $\rho = \nu \cdot p/m$ in the case $m \neq 0$), and

$$h \in H_{p_0,\rho} \equiv \{(h_0 \in H | h_0 \cdot p_0 = p_0, \ h_0 \cdot \nu_\rho = \nu_\rho\}.$$

One obtains, instead of (19), for any $h_0 \in H_{p_0,\rho}$

$$f_{\alpha,\nu_\rho}(p_0) = \pi(h_0)\overline{D_{\alpha\beta}(h_0)}f_{\beta,\nu_\rho}(p_0) \tag{46}$$

(for $N = 4$ $H_{p_0,\rho} \simeq SU(2)$, and for $N = 3, H_{p_0,\rho} \simeq \mathbf{Z}$) and one can write a theorem of the same type as theorem 1.

Let us note in closing that the construction of the free fields based on Weinberg anszatz is extremely sensitive to the choice of the configuration space X. For instance, let us take (instead of (42)) X to be the manifold of straight lines in the Minkowski space M_N. This space can be obtained from (41) factorizing to the equivalence relation $(x, \nu) \sim (x', \nu') \iff \nu = \nu', \ x - x'$ is parallel to ν. We will get something of the type (45) with $(x, \nu) \mapsto [x, \nu]$ (i.e. the equivalence class of (x, ν)) and one easily obtains $f_{\alpha,[x,\nu]} = 0$. So, in this case we cannot construct free fields.

References

[1] S. Weinberg, "Feynman Rules for Any Spin", Phys. Rev. 133 (1964) B1318-1332

[2] S. Weinberg, "Feynman Rules for Any Spin II. Massless Particles", Phys. Rev. 134 (1964) B882-896

[3] S. Weinberg, "Feynman Rules for Any Spin III", Phys. Rev. 181 (1969) 1893-1899

[4] S. Weinberg, "The Quantum Theory of Massless Particles", Brandeis Lectures, 1964, notes by D. Weaver, Prentice-Hall Inc. 1965 (S. Deser, K. W. Ford, Eds.)

[5] D. R. Grigore, "Free Fields for any Spin in 1+2 Dimensions", to appear in Journ. Math. Phys., hep-th/9404028

[6] P. A. Marchetti, "Anyon Wave Functions from Anyon Field Operators", Int. Journ. Mod. Phys. A 6 (1992) 2275-2290

[7] R. F. Streater, A. S. Wightman, "*PCT, Spin and Statistics and All That*", W. A. Benjamin Inc. New York, 1964

[8] R. Jost, "*The General Theory of Quantized Fields*", AMS, Providence, 1965

[9] N. N. Bogoliubov, A. A. Logunov, I. T. Todorov, "*Axiomatic Quantum Field Theory*", W. A. Benjamin Inc., New York, 1975

[10] V. S. Varadarajan, *Geometry of Quantum Theory*" (second edition), Springer, 1985

[11] M. Reed, B. Simon, *Methods of Modern Mathematical Physics, vol II: Fourier Analysis, Self-Adjointness*", Academic Press, 1975

[12] D. R. Grigore, "The Projective Unitary Irreducible Representations of the Poincaré Group in 1+2 Dimensions", Journ. Math. Phys. 34 (1993) 4172-4189, hep-th/9304142

[13] K. Baumann, "Quantum Fields in 1+1-Dimensions Carrying a True Ray Representation of the Poincaré Group", preprint Götingen, 1992